

## A SURVEY ON $\Sigma$ -PRODUCTS (\*)

by

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Abstract. In this paper we give a survey on the theory of  $\Sigma$ -products and  $\Sigma_m$ -products.

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(\*) Dada la categoría y profundidad con que ha sido tratado este Survey on  $\Sigma$ -products y la carencia de buenas informaciones expositivas sobre el tema, el Comité Editorial ha considerado de gran interés incluir el trabajo en esta Sección.

## 1. Introduction.

A basic construction in General Topology is the product space of a family of topological spaces. In the study of product spaces, we are faced with some subspaces of a topological product, which one obtains fixing a point in it.

For a family  $\{X_s | s \in S\}$  of topological spaces, let  $X = \prod_{s \in S} X_s$  be the topological product and let  $p = (p_s)$  be a fixed point of  $X$ . If  $S' \subset S$ , the subspace  $P_{S'}(p) = \{x \in X | x_s = p_s \text{ for all } s \in S \setminus S'\}$  is homeomorphic to the partial product  $\prod_{s \in S'} X_s$ . If  $S$  is infinite we can consider the dense subspace  $\sigma(p) = \{x \in X | \{s | x_s \neq p_s\} \text{ is finite}\}$ .

Clearly  $\sigma(p) = \bigcup_{S' \in \mathcal{P}_f(S)} P_{S'}(p)$  (where  $\mathcal{P}_f(S)$  is the set of all finite subsets of  $S$ ). The subspace  $\sigma(p)$  is called the  $\sigma$ -product of spaces  $\{X_s | s \in S\}$  with base  $p$ . The  $\sigma$ -products have been used, for example, in the proof of the theorem of preservation of the connectedness by Cartesian products.

In relation with this notion, in 1959, H.H. Corson have introduced the  $\Sigma$ -products. For a family  $\{X_s | s \in S\}$  of topological spaces, let  $X = \prod_{s \in S} X_s$  be the topological product and let  $p = (p_s)$  be a fixed point of  $X$ , the subspace  $\Sigma(p) = \{x \in X | \{s | x_s \neq p_s\} \text{ is countable}\}$  is called the  $\Sigma$ -product of spaces  $\{X_s | s \in S\}$  with base  $p$  (H.H. Corson [1959]). Observe that, if  $p$  and  $q$  are distinct points of  $X$ , then  $\Sigma(p)$  and  $\Sigma(q)$  will not, in general, be homeomorphic..

Clearly a  $\Sigma$ -product of spaces  $\{X_s | s \in S\}$  is a proper subspace of the product space if and only if, uncountable many spaces  $X_s$  contain at least two elements; such  $\Sigma$ -products are called proper. Proper  $\Sigma$ -products are never paracompact (T.C. Przymusiński), and  $\Sigma$ -products

of compact spaces may be non normal (H.H. Corson).

We have that  $\Sigma(p) \supset \sigma(p)$ , then all  $\Sigma$ -product is dense in the product space. Moreover  $\Sigma(p) = \bigcup_{s' \in \mathcal{P}_c(S)} P_{s'}(p)$ , where  $\mathcal{P}_c(S)$  is the set of all countable subsets of  $S$ .

Before the Corson's definition, some mathematicians have studied subspaces of a Cartesian product of topological groups, which are  $\Sigma$ -products (for ex. I. Glicksberg [1959]).

Finally, in 1963 V. Efimov have introduced the  $\Sigma_m$ -products. Let  $X = \prod_{s \in S} X_s$  be a product space and  $p$  be some point in  $X$ , for each infinite cardinal  $m$ , we call  $\Sigma_m$ -product with base point  $p$  the subset  $\{x \in X \mid \text{card } \{s \in S \mid x_s \neq p_s\} \leq m\}$ . Remark that, a  $\Sigma_{\chi_0}$ -product is a  $\Sigma$ -product.

Many results have been obtained on these subjects during these years, and hitherto, there was not any survey devoted only to give a panoramic of  $\Sigma$ -products.

## 2. $\Sigma$ -products and continuous mappings.

The following theorem is very useful in the theory of topological groups.

Theorem 2.1 (I. Glicksberg [1959] and J.M. Kister [1962]). Let  $\Sigma$  be a proper  $\Sigma$ -product of compact  $T_2$  spaces  $X_s$ ,  $s \in S$ . Then  $\prod_{s \in S} X_s$  is the Stone-Čech compactification of  $\Sigma$ .

The proof of this result follows from the next fact: all  $\Sigma$ -product of compact  $T_2$  spaces is countably compact.

This theorem yields a fairly accesible body of nontrivial Stone-čech compactifications. We can also constructed, using this

result, numerous counterexamples of a homogenous space  $X$  with  $\beta X \setminus X$  not homogeneous. This subject is related with the question raised in 1955 whether the homogeneity of  $X$  implies the homogeneity of  $\beta X \setminus X$  and studied by W. Rudin in [1956]. He showed that  $\beta X \setminus X$  fails to be homogeneous whenever  $X$  is locally compact but not sequentially compact.

Theorem 2.1 has been rediscovered by J.M. Kister in [1962] as a source of counterexamples of a non-discrete noncompact topological group such that every real-valued function on it, is uniformly continuous.

Finally this theorem yields as a corollary that, every continuous mapping  $f: \Sigma \rightarrow Y$  of a  $\Sigma$ -product of compact  $T_2$  spaces  $X_s$ ,  $s \in S$  to a compact space  $Y$ , is extendable to a mapping  $F: \prod_{s \in S} X_s \rightarrow Y$ .

On the real-compactification of a  $\Sigma$ -product, Corson established also a theorem.

Theorem 2.2 (H.H. Corson [1959]) If  $\Sigma$  is a  $\Sigma$ -product of separable metrizable spaces  $X_s$ ,  $s \in S$ , then  $\prod_{s \in S} X_s$  is the real-compactification of  $\Sigma$ .

From this theorem it follows that, every continuous real-valued function  $f: \Sigma \rightarrow \mathbb{R}$  of a  $\Sigma$ -product of separable metrizable spaces  $X_s$ ,  $s \in S$ , is extendable to a continuous function  $\tilde{f}: \prod_{s \in S} X_s \rightarrow \mathbb{R}$ .

In 1966, R. Engelking established the following theorems on extension of mappings defined on a  $\Sigma$ -product. The first result generalizes the above theorems on compactifications.

Theorem 2.3 (R. Engelking [1966]) If  $\{X_s\}_{s \in S}$  is a family of  $T_1$  spaces such that every finite product has the Lindelöf property and  $Y$  is a  $T_2$  space such that the diagonal of  $Y \times Y$  is a  $G_\delta$ -set, then for any

$p \in \prod_{s \in S} X_s$  and a continuous function  $f$  from the  $\Sigma$ -product with base  $p$ ,  $\Sigma$ , to  $Y$ , there exists a factorization  $f = f' \circ \pi_{S'}|_{\Sigma}$ , where  $S' \subset S$  is a countable set and  $f'$  is a continuous function from  $\prod_{s \in S'} X_s$  to  $Y$ . In particular  $f$  is in this case extendable to the product  $\prod_{s \in S} X_s$ .

Theorem 2.4 (R. Engelking [1966]) If  $\Sigma$  is a  $\Sigma$ -product of a family  $X_s$ ,  $s \in S$  of  $T_1$  separable, first-countable spaces, such that every finite product of them is Lindelöf, and  $f: \Sigma \rightarrow Y$  is a sequentially continuous function from  $\Sigma$  to a  $T_2$  space  $Y$ , in which every point is a sequentially  $G_\delta$ -set, then there exists a factorization,  $f = f' \circ \pi_{S'}|_{\Sigma}$ , where  $S' \subset S$  is countable and  $f'$  is a continuous function from  $\prod_{s \in S'} X_s$  to  $Y$ .

Theorem 2.5 (R. Engelking [1966]) If  $A$  is a subset of the Cartesian product  $\prod_{s \in S} X_s$  of second-countable  $T_1$  spaces, where  $\text{card } S \leq$  the first inaccessible cardinal, invariant under composition, and  $f: A \rightarrow Y$  is a sequentially continuous function from  $A$  to a  $T_2$  space  $Y$  in which every point is a sequentially  $G_\delta$ -set, then there exists a factorization  $f = f' \circ \pi_{S'}|_A$ , where  $S' \subset S$  is countable and  $f'$  is a continuous function from  $\prod_{s \in S'} X_s(A)$  to  $Y$ . Moreover if a  $\Sigma$ -product with base  $p$  is contained in  $A$ , then it suffices to suppose that  $x(T, p) \in A$  for  $x \in A$  and  $T \subset S$ ,  $X_s$ 's are  $T_1$  separable, first-countable, and every finite product of them is Lindelöf.

It is interesting to study  $\Sigma$ -products of metrizable spaces. In 1977, S.P. Gul'ko proved the following result which solves a problem of Corson.

Theorem 2.6 (S.P. Gul'ko [1977]) If  $\Sigma$  is a  $\Sigma$ -product of metrizable spaces, then  $\Sigma$  is normal (and collectionwise normal).

This result have been obtained also by M.E. Rudin. In the Gul'ko proof, let  $F_1$  and  $F_2$  be disjoint closed subsets of  $\Sigma$ , he constructs a locally finite covering  $\mathcal{C}$  of  $\Sigma$  consisting of sets whose closures are disjoint either from  $F_1$  or from  $F_2$ , then the sets  $U_j = \Sigma \setminus \{\bar{C} \mid C \in \mathcal{C}, \bar{C} \cap F_j \neq \emptyset\}$  are disjoint neighborhoods of the sets  $F_j$ ,  $j=1,2$ . Analogously for collectionwise normal.

The following theorem generalizes the above theorem on metrizable spaces.

Theorem 2.7 (A.P. Kombarov [1978]) If  $\Sigma$  is proper  $\Sigma$ -product of paracompact  $p$ -spaces, then the following are equivalent:

1.  $\Sigma$  has countable tightness.
2.  $\Sigma$  is collectionwise normal.
3.  $\Sigma$  is normal.

Theorem 2.8 (H.H. Corson [1959]) Every metrizable space can be imbedded as a subspace of a  $\Sigma$ -product of copies of the unit interval.

The last theorem has motived the study of spaces which are embeddable in  $\Sigma$ -products of copies of the real line. We see this in the next section.

Theorem 2.9 (H.H. Corson [1959]) Let  $\Sigma$  be a  $\Sigma$ -product of copies of the real line. Then  $\Sigma$  is homeomorphic to  $C(X)$ , all the continuous real functions on a Lindelöf space  $X$ , where  $C(X)$  has the simple topology (or the compact-open topology).

Theorem 2.10 (S.P. Gul'ko [1977], and K. Alster and R. Pol [1980]) The space  $C(X,M)$  is Lindelöf under the pointwise topology when  $X$  is a closed subset of a  $\Sigma$ -product of metrizable separable spaces and  $M$  is metrizable and separable.

Recall that: a map  $f: \prod_{s \in S} X_s \rightarrow Z$  is  $\Sigma$ -continuous if its restriction

to each  $\Sigma$ -product of  $X_s$ 's is continuous, and a map  $f: \prod_{s \in S} X_s \rightarrow Z$  is  $\Sigma$ -semicontinuous if whenever  $U$  is open and  $x \in f^{-1}(U)$ , there exists a finite  $F \subset S$  such that  $\Sigma_F(x) \subset f^{-1}(U)$ , where  $\Sigma_F(x) = \{y \mid \{s \mid x_s \neq y_s\} \text{ is countable and disjoint from } F\}$ . (N. Noble [1970]).

Theorem 2.11 (N. Noble [1970]) Let  $G = \prod_{s \in S} G_s$  where  $G_s$  is a topological group, let  $H$  be a subgroup of  $G$  which is invariant under projections, and let  $f$  be a homomorphism on  $H$  with  $T_0$  range. If  $f$  is continuous on each factor and  $\Sigma$ -semicontinuous, then  $f$  is  $\Sigma$ -continuous.

Theorem 2.12 (N. Noble [1970]) Each sequentially continuous function on  $\prod_{s \in S} X_s$  is  $\Sigma$ -semicontinuous.

### 3. Subsets of $\Sigma$ -products.

Clearly all  $\sigma$ -product with base point  $p$  is contained in the  $\Sigma$ -product with base point  $p$ . Nevertheless, in this survey we shall not include the results on these subspaces, but only those results strictly on  $\Sigma$ -products.

Corson's theorem on embedding of metrizable spaces in  $\Sigma$ -products (Theorem 2.7) motives the notion of Corson compact: Every  $T_2$  compact space that is homeomorphic to some closed subset of a  $\Sigma$ -product of copies of the real line is called a Corson compact (E. Michael and M.E. Rudin [1977]).

Theorem 3.1 (V. Efimov and G.I. Čertanov [1978], and M.E. Rudin) A Corson compact linealy ordered space is metrizable.

Theorem 3.2 (S.P. Gul'ko [1977]) The closed image of a Corson compact is Corson compact.

Theorem 3.3 (E. Michael and M.E. Rudin [1977]) Let  $X$  be a compact

$T_2$  space. Then  $X$  is Corson compact if and only if  $X$  has a point-countable  $T_0$ -separating cover by open  $F_\sigma$ -sets (Recall that a collection  $\mathcal{U}$  of subsets of  $X$  is  $T_0$ -separating if whenever  $x, y \in X$  then, some  $U \in \mathcal{U}$  contains exactly one of  $x$  and  $y$ ).

Theorem 3.4 (N.N. Yakovlev [1980]) Every Corson compact is hereditarily metalindelöf.

This theorem is a corollary of the result which asserts that every compact subset of a  $\Sigma$ -product of copies of the unit interval is hereditarily metalindelöf.

Theorem 3.5 (N.N. Yakovlev [1980]) Let  $X$  be a scattered compact  $T_2$ , then  $X$  is hereditarily metalindelöf if and only if,  $X$  is a Corson compact.

Theorem 3.6 (N.N. Yakovlev [1980]). Every scattered Corson compact admits a closure preserving converging by countable compacts.

Theorem 3.7 (G. Gruenhage [1984]) Let  $X$  be a compact  $T_2$  space and let  $\Delta$  be the diagonal of  $X \times X$ , then the following are equivalent:

- (a)  $X$  is Corson compact.
- (b)  $X^2 \setminus \Delta$  is metalindelöf.
- (c)  $X^2$  is hereditarily metalindelöf.

Theorem 3.8 (G. Gruenhage [1984]) A compact  $T_2$  space is Corson compact if and only if the diagonal  $\Delta$  is a  $W$ -set in  $X^2$ .

Theorem 3.9 (R.J. Gardner [1987]) A universally measure zero Corson compact is hereditarily metacompact.

Theorem 3.10 (W. Marciszewski [1991]). For Corson compact with weight  $> \omega_1$ , then spread is equal to weight.



#### 4. $\Sigma_m$ -products.

The notion of  $\Sigma_m$ -products has been introduced by V. Efimov in [1963] as a generalization of  $\Sigma$ -products for each infinite cardinal  $m$ .

Theorem 4.1 (V. Efimov [1963]) The weight of a compact  $T_2$  space that is the continuous image of a  $\Sigma_m$ -product of compact  $T_2$  spaces  $X_s$  of weight  $m_s \geq 2$  does not exceed  $\max \{m, \sup m_s\}$ .

Remark (A.P. Kombarov [1973]) If  $m > \chi_0$  and if an uncountable number of paracompact spaces  $X_s$  are not compact, then a  $\Sigma_m$ -product of  $X_s$ 's is not normal.

Theorem 4.2 (A.P. Kombarov and V.I. Malyhin [1973]) If the tightness of any finite product of spaces  $X_s$ ,  $s \in S$  does not exceed  $m$ , then the tightness of a  $\Sigma_m$ -product of the spaces  $X_s$  does not exceed  $m$ .

Theorem 4.3 (A.P. Kombarov [1978]) Let  $\Sigma_m$  be a  $\Sigma_m$ -product of compact  $T_2$  spaces. Then the following are equivalent:

- 1:  $\Sigma_m$  has tightness no greater than  $m$ .
2.  $\Sigma_m$  is normal.

Theorem 4.4 (M. Ulmer [1973]) Let  $\Sigma_m$  be a  $\Sigma_m$ -product of  $p$ -spaces, then  $\Sigma_m$  is  $C$ -embedded in the Cartesian product.

#### 5. The weak $\mathcal{B}$ -property and $\Sigma$ -products.

Let  $\mathcal{F} = \{F_s \mid s \in S\}$  be a collection of subsets of a space  $X$ .  $\mathcal{F}$  is said to be monotone decreasing if  $S$  is well-ordered and  $F_s \supset F_{s'}$ , for each  $s, s' \in S$  with  $s \leq s'$ . The space  $X$  is said to have the weak

$\mathcal{B}$ -property if for each monotone decreasing family  $\{F_s | s \in S\}$  of closed sets of  $X$  with  $\bigcap_{s \in S} F_s = \emptyset$ , there is a family  $\{U_s | s \in S\}$  of open sets of  $X$  such that  $\bigcap_{s \in S} \bar{U}_s = \emptyset$  and  $U_s \supset F_s$  for each  $s \in S$  (Y. Yasui).

The notion of the weak  $\mathcal{B}$ -property is connected with normality of product spaces, then the problem of preservation of this property by  $\Sigma$ -products can be studied.

Theorem 5.1 (K. Chiba [1982]) Every  $\Sigma$ -product of compact spaces has the weak  $\mathcal{B}$ -property.

Theorem 5.2 (K. Chiba [1982]) Every  $\Sigma$ -product of Lindelöf  $M$ -spaces of countable tightness has the weak  $\mathcal{B}$ -property.

Theorem 5.3 (K. Chiba [1982]) Every  $\Sigma$ -product of separable metrizable spaces has the weak  $\mathcal{B}$ -property.

Theorem 5.4 (K. Chiba [1982], [1985]) Suppose that every countable product of spaces  $X_s$  is Lindelöf, let  $\Sigma$  be a  $\Sigma$ -product of  $X_s$ . Then  $\Sigma$  is countably paracompact if and only if  $\Sigma$  has the weak  $\mathcal{B}$ -property.

Theorem 5.5 (K. Chiba [1982]) Let  $\Sigma$  be a  $\Sigma$ -product of  $X_s$ ,  $s < \omega_1$ , where each  $X_s$  is metrizable, then  $\Sigma$  has the weak  $\mathcal{B}$ -property.

## 6. The shrinking property and $\Sigma$ -products.

In 1942, S. Lefschetz established the important characterization of normal spaces by shrinking of point-finite open coverings: Let  $X$  be a topological space, then  $X$  is normal if and only if for every

point-finite open covering  $\{U_s\}_{s \in S}$  of  $X$  there exists an open covering  $\{V_s\}_{s \in S}$  of  $X$  such that  $\bar{V}_s \subset U_s$  for every  $s \in S$ . This result motivates the following definition: A space has the shrinking property if, for every open covering  $\{U_s\}_{s \in S}$  there is an open covering  $\{V_s\}_{s \in S}$  with  $\bar{V}_s \subset U_s$  for each  $s \in S$  (M.E. Rudin [1983]). The problem of preservation of the shrinking property by  $\Sigma$ -products has been studied by many authors.

Theorem 6.1 (M.E. Rudin [1983]) A  $\Sigma$ -product of metric spaces has the shrinking property.

Theorem 6.2 (A. Le Donne [1985]) A  $\Sigma$ -product of paracompact  $p$ -spaces with countable tightness has the shrinking property.

Recall that a space  $X$  is said to be a  $\Sigma$ -space if there are a  $\sigma$ -locally finite closed covering  $\mathcal{F}$  of  $X$  and a covering  $\mathcal{C}$  of  $X$  by countably compact sets such that, whenever  $C \in \mathcal{C}$  and  $U$  is open in  $X$  with  $C \subset U$ , then  $C \subset F \subset U$  for some  $F \in \mathcal{F}$  (K. Nagami).

Theorem 6.3 (Y. Yajima [1984 a]) Let  $\Sigma$  be a  $\Sigma$ -product of paracompact  $\Sigma$ -spaces. If  $\Sigma$  has countable tightness, then it is collectionwise normal.

Theorem 6.4 (Y. Yajima [1984 a]) Let  $\Sigma_m$  be a  $\Sigma_m$ -product of paracompact  $\Sigma$ -spaces  $X_s$ ,  $s \in S$ . If  $\Sigma_m$  has tightness  $\leq m$ , then the following are equivalent:

- (a)  $\prod_{s \in S'} X_s$  is normal for each  $S' \subset S$  with  $\text{card } S' \leq m$ .
- (b)  $\Sigma_m$  is (collectionwise) normal.

Theorem 6.5 (Y. Yajima [1984 b], [1986], [1989]) Let  $\Sigma$  be a  $\Sigma$ -product of paracompact  $\Sigma$ -spaces. Then the following are equivalent:

- (a)  $\Sigma$  is collectionwise normal
- (b)  $\Sigma$  is normal

(c)  $\Sigma$  has the shrinking property.

Theorem 6.6 (Y. Yajima [1986] [1989]) A  $\Sigma$ -product of paracompact  $M$ -spaces has the weak  $B$ -property.

(This theorem extends some results of K. Chiba, cited in Section 5).

## 7. $\Sigma$ -products of semi-stratifiable spaces.

A space  $X$  is said to be semi-stratifiable if there is a function  $g$  of  $X \times \mathbb{N}$  into the topology of  $X$  satisfying

(i)  $\bigcap_{n \in \mathbb{N}} g(x, n) = \{x\}$  for each  $x \in X$

(ii) if  $(x_n)$  is a sequence of points in  $X$  with  $y \in \bigcap_{n \in \mathbb{N}} g(x_n, n)$  for

some  $y \in X$ , then  $(x_n)$  converges to  $y$  (G.D. Creede).

Theorem 7.1 (Y. Yajima [1987]) Let  $\Sigma$  be a  $\Sigma$ -product of semi-stratifiable spaces, each finite products of which is paracompact. If  $\Sigma$  has countable tightness, then it is normal.

Theorem 7.2 (Y. Yajima [1987]) Let  $\Sigma$  be a  $\Sigma$ -product of semi-stratifiable spaces, each finite product of which is paracompact. Then, if  $\Sigma$  is normal, it is also collectionwise normal.

In 1975, M. Morishita raised the question of if there is a non-normal  $\Sigma$ -product, each countable product of which is paracompact and has countable tightness. This problem has been affirmatively answered by T. Daniel and G. Gruenhage [1992] who constructed a such space.

A space  $X$  is said to be  $\delta$ -normal if for any disjoint closed sets  $A$  and  $B$  there are disjoint  $G_\delta$ -sets  $G$  and  $H$  such that  $A \subset G$  and  $B \subset H$  (J. Chaber).  $X$  is collectionwise  $\delta$ -normal if whenever  $\{F_s \mid s \in S\}$  is a discrete collection of closed sets in  $X$  there is a collection

$\{H_s | s \in S\}$  of pairwise disjoint  $G_\delta$ -sets such that  $F_s \subset G_s$  for every  $s \in S$  (H.J.K. Junnila).

Theorem 7.3 (H. Teng [1991]) A  $\Sigma$ -product of semi-stratifiable spaces is collectionwise  $\delta$ -normal.

Theorem 7.4 (Y. Yajima [1987]), A  $\Sigma$ -product of semi-stratifiable spaces is a Morita's P-space.

Theorem 7.5 (Y. Yajima [1987]) Any normal proper  $\Sigma$ -product is countably paracompact.

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